

CS 61A

DISCUSSION 4

LIST MUTATION, ORDERS OF GROWTH, AND NONLOCAL

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Discussion 134
UC Berkeley Fall 16

AGENDA

- Announcements
- List Mutation
- Orders of Growth
- Nonlocal
- Appendix
 - Dictionaries

ANNOUNCEMENTS

- Maps due tonight
- Lab 5 due Friday 9/30
- CSM signups reopened
- 61A one-on-one tutoring

CHALLENGE QUESTION

- For those who runs through the packet, what is the order of the growth for the function below?

```
def f(n):  
    i = 2  
    while i < n:  
        print(i)  
        i = i * i
```


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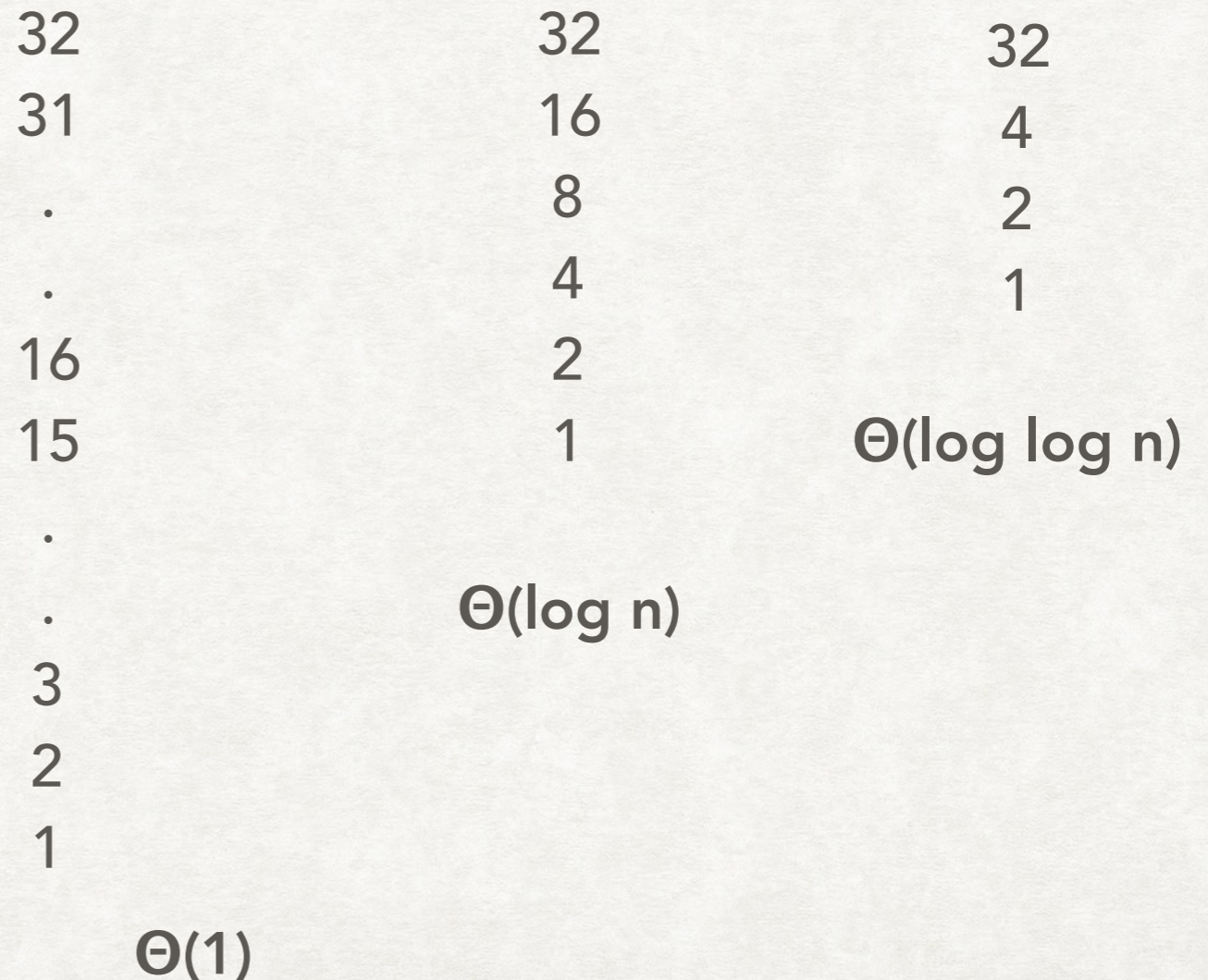
$\Theta(\log \log n)$

CHALLENGE QUESTION

- For those who runs through the packet, what is the order of the growth for the function below?

```
def f(n):  
    i = 2  
    while i < n:  
        print(i)  
        i = i * i
```

$\Theta(\log \log n)$



MUTATION

- When we define functions, we created function objects in environment diagrams.
- When we create lists, we create list objects.
- We can change the elements of list objects after we've created it.

```
>>> a = [1, 2, 3]
```

```
>>> a
```

```
[1, 2, 3]
```

```
>>> a[2] = 100
```

```
>>> a
```


MUTATION

- When we define functions, we created function objects in environment diagrams.
- When we create lists, we create list objects.
- We can change the elements of list objects after we've created it.

```
>>> a = [1, 2, 3]
```

```
>>> a
```

```
[1, 2, 3]
```

```
>>> a[2] = 100
```

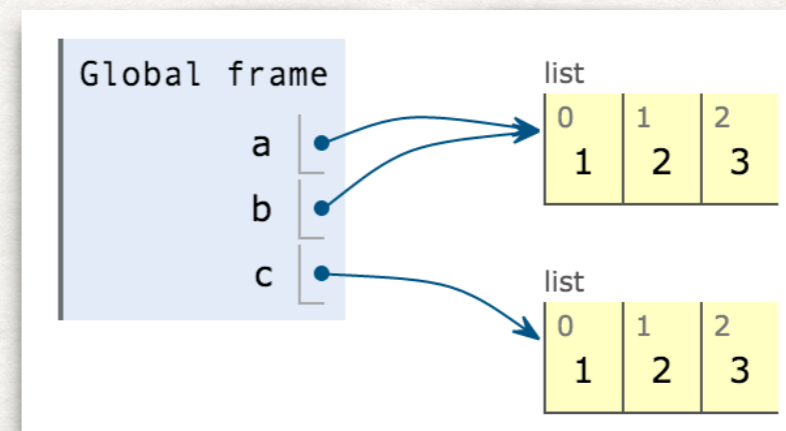
```
>>> a
```

```
[1, 2, 100]
```


MUTATION

- If I assign this variable a to variable b, b receives the reference.
- a and b is the same list as they are both referencing the same list object
- a, b, and c have the same elements, but a and c are not the same list

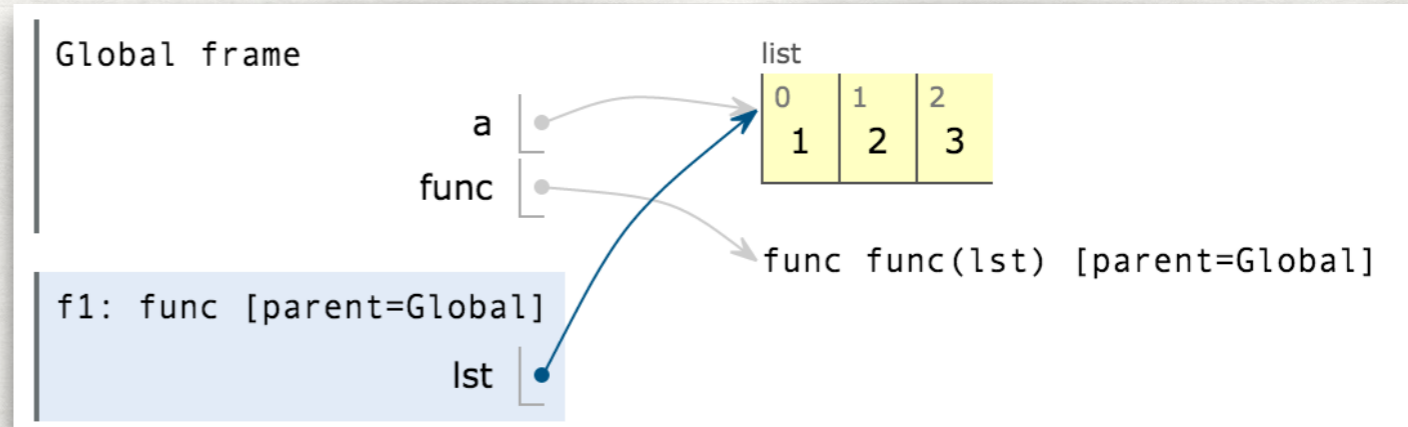
```
1 a = [1, 2, 3]
2 b = a
3 c = [1, 2, 3]
```



MUTATION

- When we assign a list to a variable, the variable references the list object.
- If I pass in a variable that references a list to a function argument, I am passing in the reference.
- This is similar to passing in a function object.

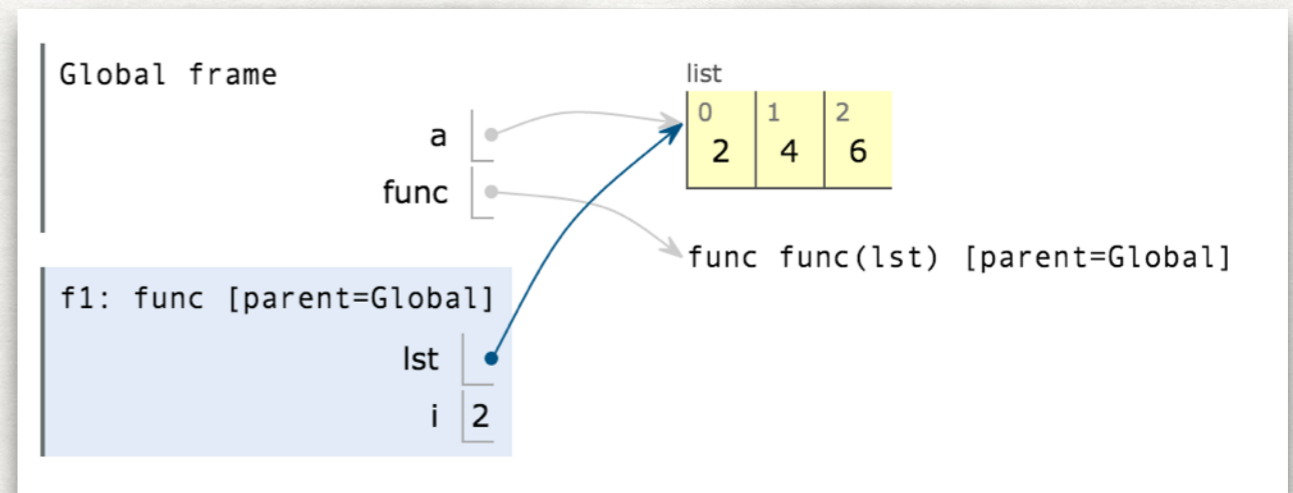
```
1 a = [1, 2, 3]
→ 2 def func(lst):
3     for i in range(0, len(lst)):
4         lst[i] = lst[i] * 2
5
→ 6 func(a)
```



MUTATION

- Within the body of `func`, `lst`'s values are changed. Notice that `a`'s values are also changed because `lst` references the same list `a` is pointing to.

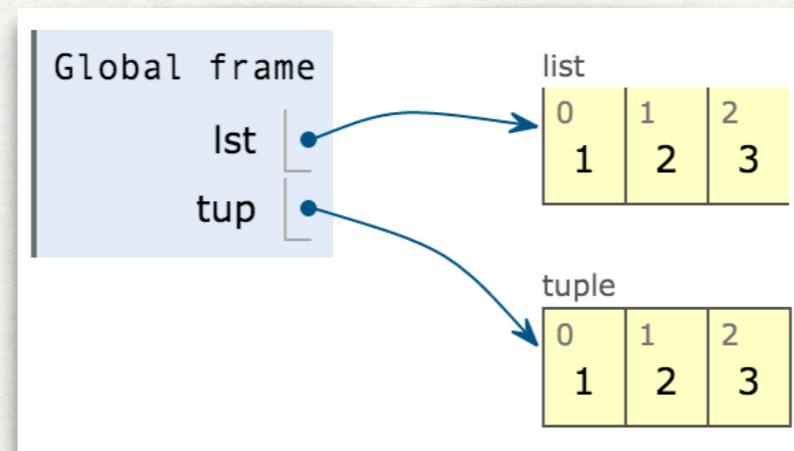
```
1 a = [1, 2, 3]
2 def func(lst):
3     for i in range(0, len(lst)):
4         lst[i] = lst[i] * 2
5
6 func(a)
```



MUTATION

- Lists and dictionaries are mutable.
- Tuples and strings are immutable. Once they are created, they cannot be changed.

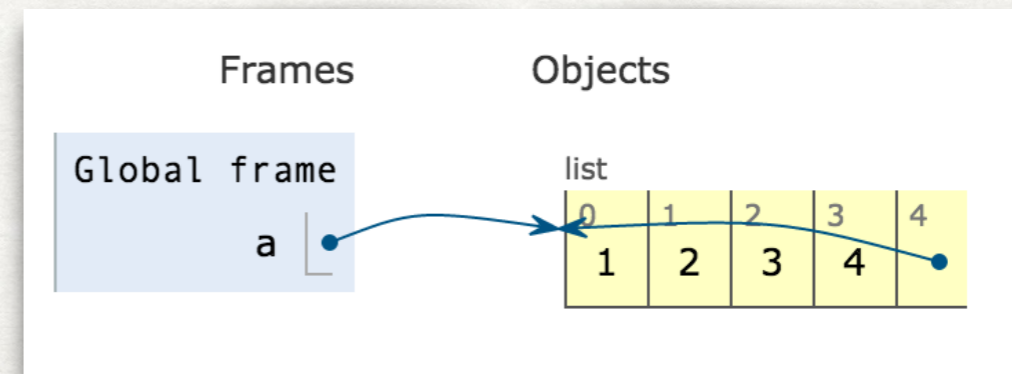
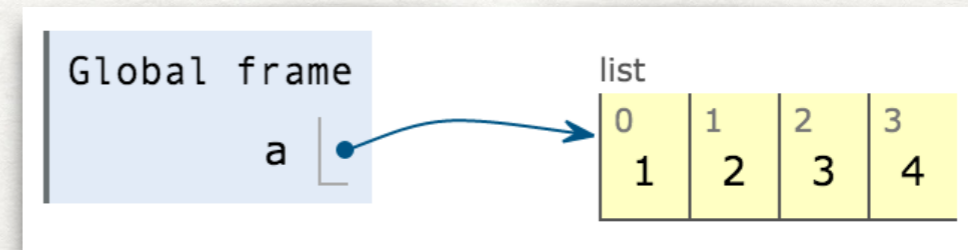
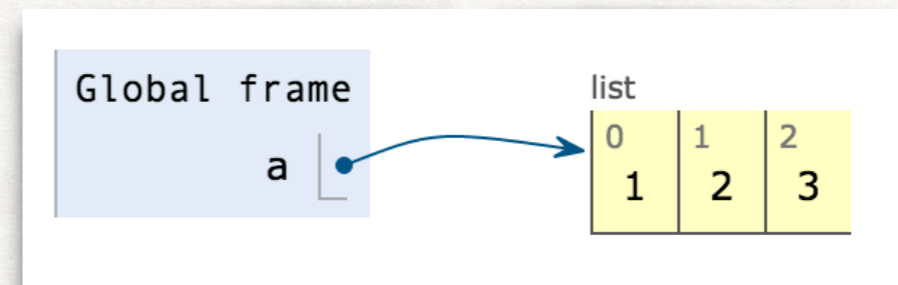
```
1 lst = [1, 2, 3]
→ 2 tup = (1, 2, 3)
```



MUTATION

- `lst.append(x)` adds `x` to the end of the list.
- Only creates one new index.

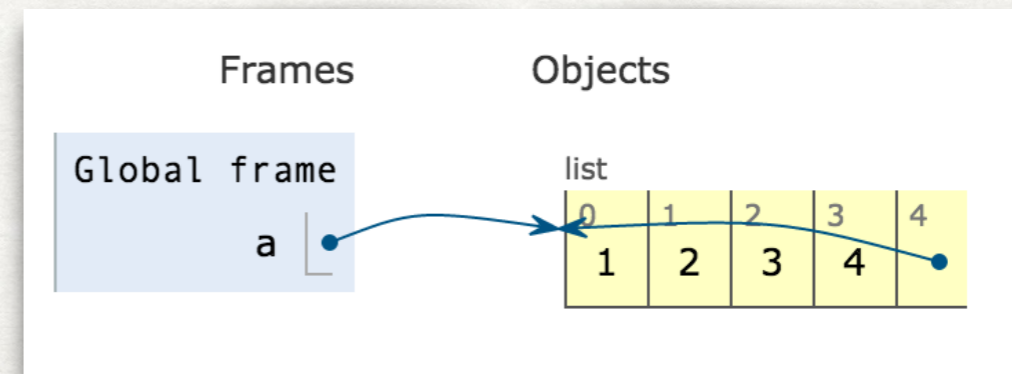
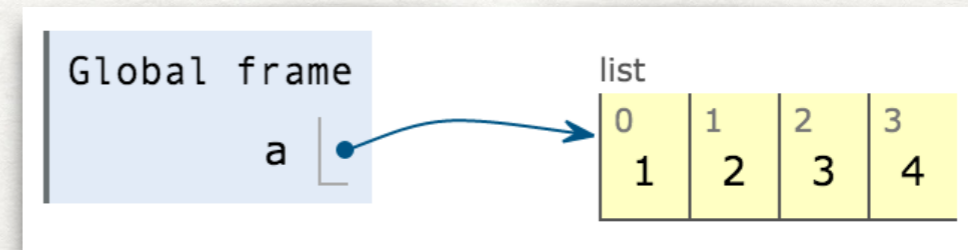
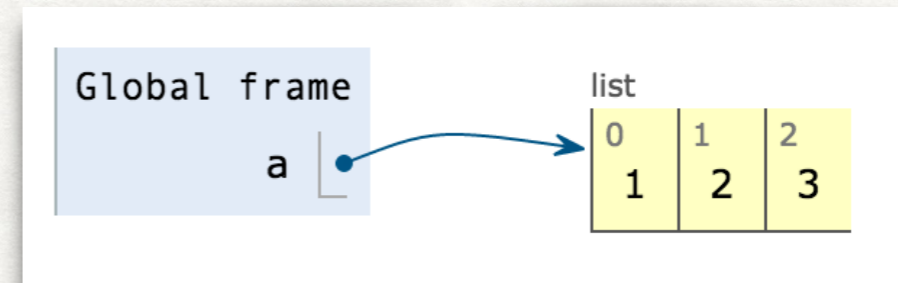
```
>>> a = [1, 2, 3]
>>> a.append(4)
>>> a
[1, 2, 3, 4]
>>> a.append([5, 6])
>>> a
[1, 2, 3, 4, [5, 6]]
>>> len(a)
5
```



MUTATION

- A list can append itself.

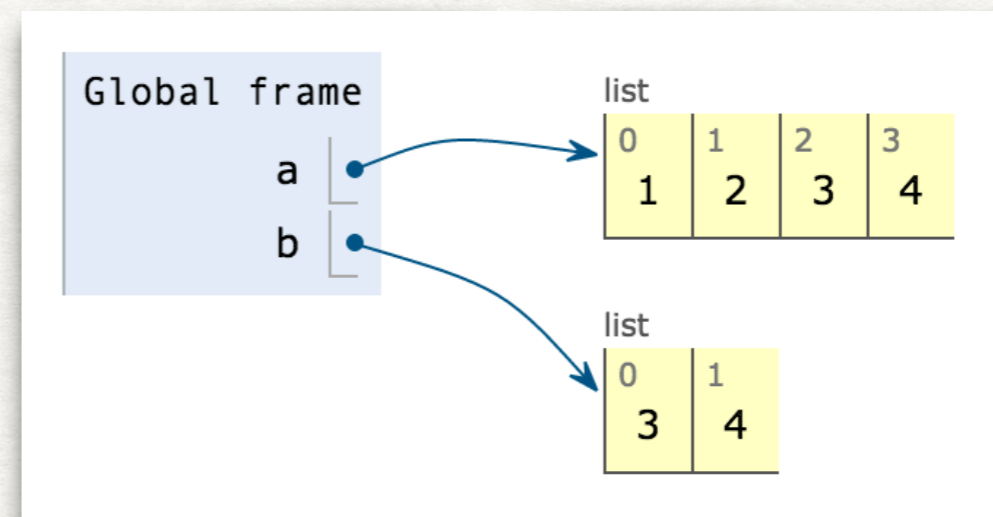
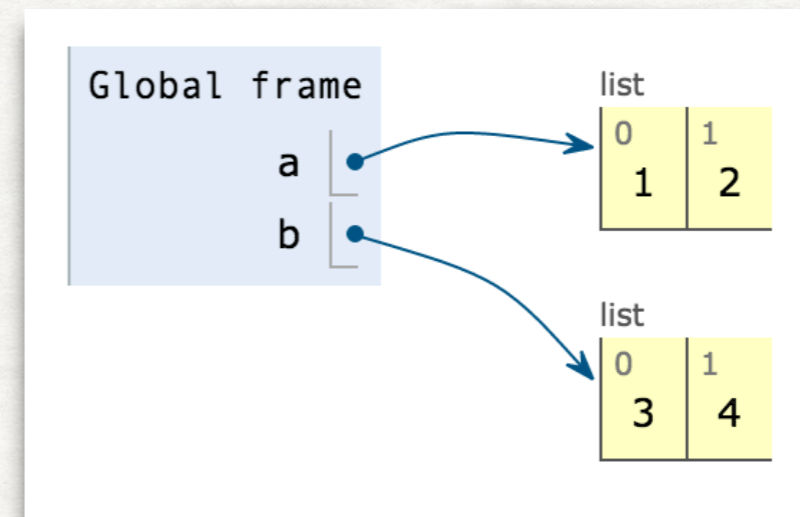
```
>>> a = [1, 2, 3, 4]
>>> a.append(a)
>>> a
[1, 2, 3, 4, [...]]
>>> a[4][3]
4
>>> a[4][4][4][2]
3
```



MUTATION

- `lst.extend(seq)` appends each element of `seq` to `list`.
- `seq` can be a list or a tuple.
- tinyurl.com/mutation-q1

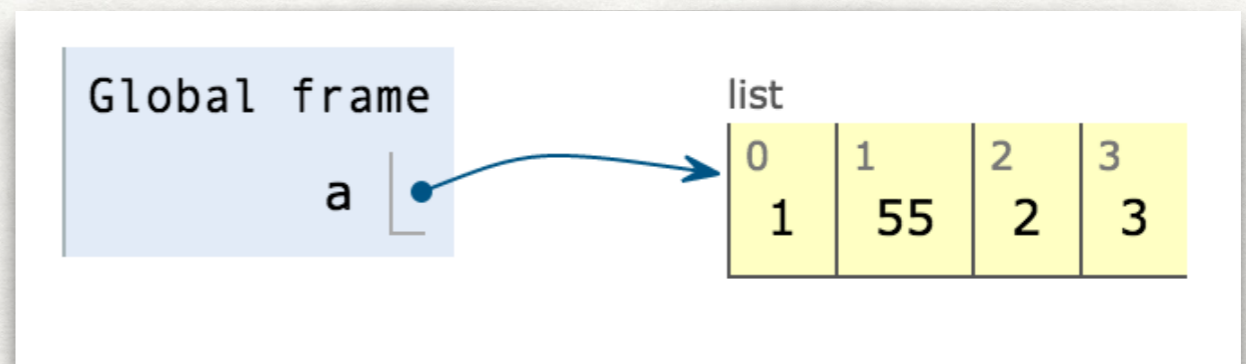
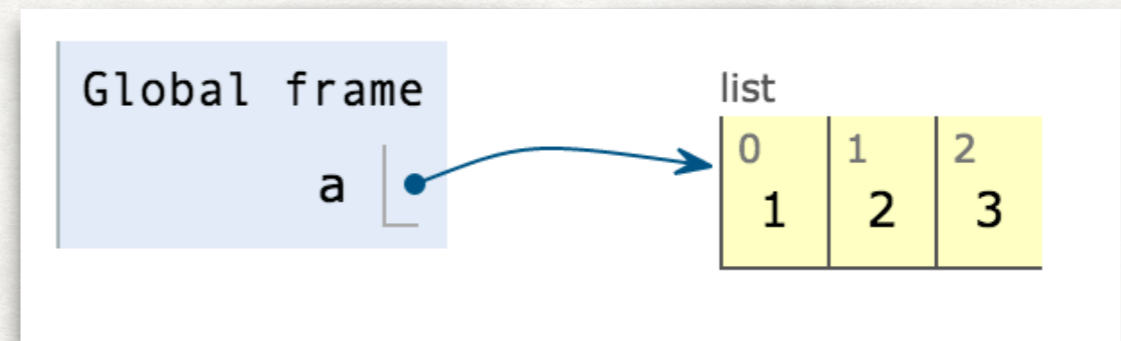
```
>>> a = [1, 2]
>>> b = [3, 4]
>>> a.extend(b)
>>> a
[1, 2, 3, 4]
>>> b
[3, 4]
```



MUTATION

- `lst.insert(i, x)` inserts `x` at index `i` by adding a new element and not replace the original element at `i`.

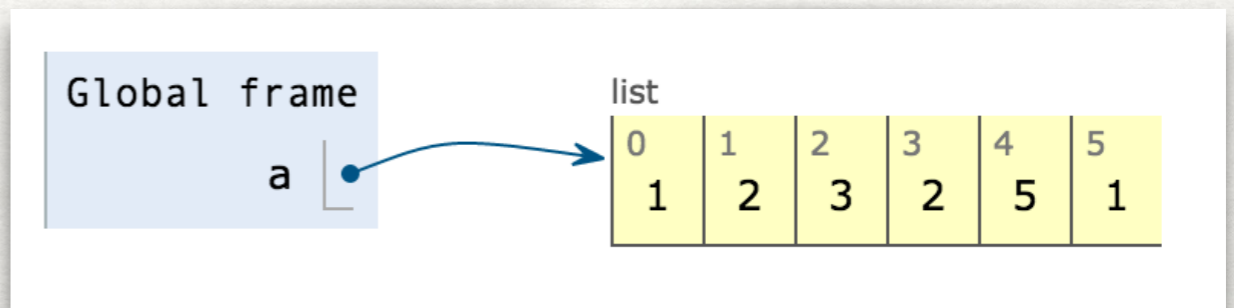
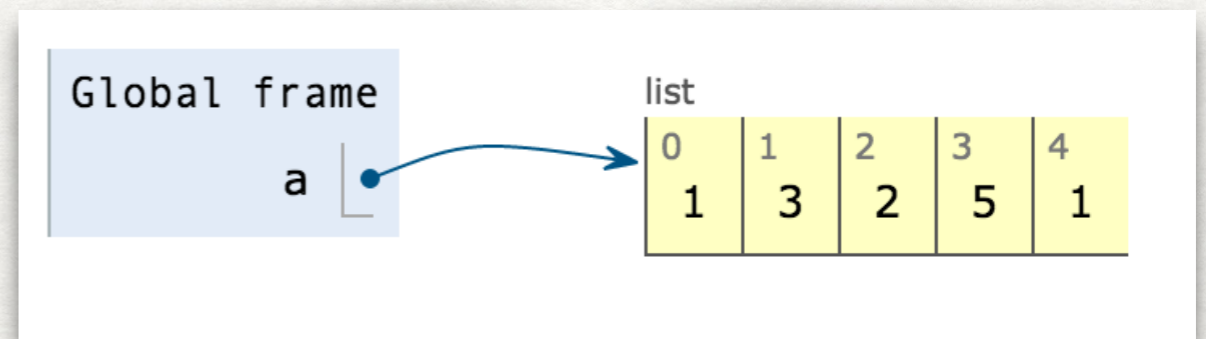
```
>>> a = [1, 2, 3]
>>> a.insert(1, 55)
>>> a
[1, 55, 2, 3]
```



MUTATION

- `lst.remove(x)` removes the first time we see `x` in a list, otherwise errors

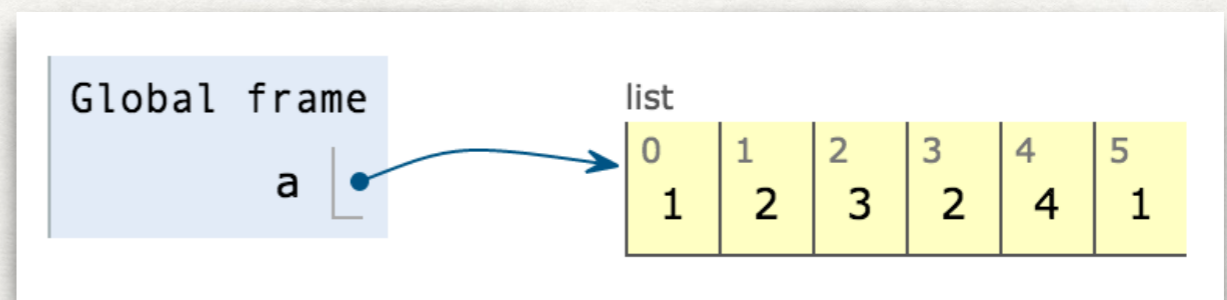
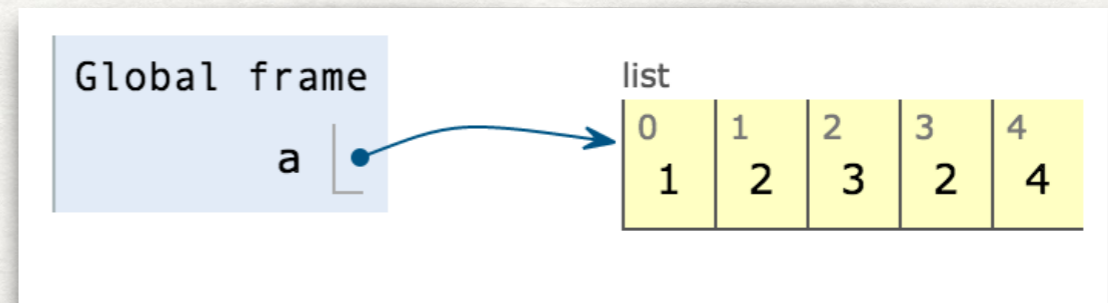
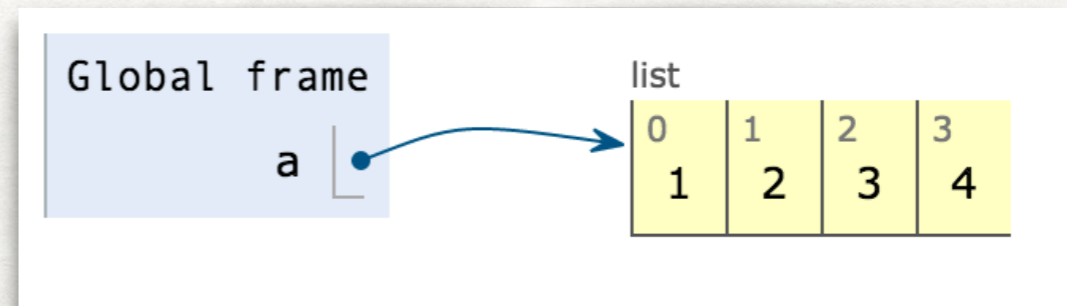
```
>>> a = [1, 2, 3, 2, 5, 1]
>>> a.remove(2)
>>> a
[1, 3, 2, 5, 1]
```



MUTATION

- `lst.pop(i)` removes and returns the element at index `i`. By default, `i` is the last element.

```
>>> a = [1, 2, 3, 2, 4, 1]
>>> a.pop()
1
>>> a.pop(3)
2
>>> a
[1, 2, 3, 4]
```



MUTATION

- `+=` for lists mutates the original list.
- `+=` is different from `a = a + [1]` when `a` is a list.
- Evaluating right hand side creates a new list and then assigns the new list to `a`.

```
>>> a = [1, 2, 3, 4]
>>> id(a)
<some id 1>
>>> a += [3]
>>> a
[1, 2, 3, 4, 3]
>>> id(a)
<some id 1>
```

```
>>> a = a + [2]
>>> a
[1, 2, 3, 4, 3, 2]
>>> id(a)
>>>
<some id 2>
```


MUTATION

- `+=` for lists mutates the original list, but is still a “reassignment”.
- Thus the list needs to be in the local frame.
- Using `append` or `extend` only require access to the list.
- It can be in the parent frame.

MUTATION

```
lst = [1, 2, 3]
def f():
    lst.append(4)

f()
print(lst)
```

[1, 2, 3, 4]

```
def g():
    lst += [5]

g()
```

Error

MUTATION

```
>>> lst1 = [1, 2, 3]
```

```
>>> lst2 = [1, 2, 3]
```

```
#compares each value
```

```
>>> lst1 == lst2
```

```
#compares references
```

```
>>> lst1 is lst2
```

```
>>> lst2 = lst1
```

```
>>> lst2 is lst1
```

```
>>> lst1.append(4)
```

```
>>> lst1
```

```
>>> lst2
```

```
>>> lst2[1] = 42
```

```
>>> lst2
```

```
>>> lst1 = lst1 + [5]
```

```
>>> lst1 == lst2
```

```
>>> lst1
```

```
>>> lst2
```

```
>>> lst2 is lst1
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MUTATION

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>>> lst1 = [1, 2, 3]
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```

```
True
```

```
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```

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```

```
>>> lst2
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```
>>> lst2 is lst1
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False
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>>> lst1.append(4)
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>>> lst1
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>>> lst2
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```

```
>>> lst1.append(4)
```

```
>>> lst1
```

```
[1, 2, 3, 4]
```

```
>>> lst2
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False
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```
>>> lst2 is lst1
```

```
True
```

```
>>> lst1.append(4)
```

```
>>> lst1
```

```
[1, 2, 3, 4]
```

```
>>> lst2
```

```
[1, 2, 3, 4]
```

```
>>> lst2[1] = 42
```

```
>>> lst2
```

```
[1, 42, 3, 4]
```

```
>>> lst1 = lst1 + [5]
```

```
>>> lst1 == lst2
```

```
>>> lst1
```

```
>>> lst2
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MUTATION

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```
>>> lst1.append(4)
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```
>>> lst1
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```
[1, 2, 3, 4]
```

```
>>> lst2
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[1, 2, 3, 4]
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>>> lst2[1] = 42
```

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>>> lst2
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```
>>> lst1 = lst1 + [5]
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>>> lst1 == lst2
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False
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```
>>> lst1
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```
>>> lst2
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>>> lst2 is lst1
```


MUTATION

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>>> lst1 == lst2
True
#compares references
>>> lst1 is lst2
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>>> lst2 is lst1
True
>>> lst1.append(4)
>>> lst1
[1, 2, 3, 4]
>>> lst2
[1, 2, 3, 4]
```

```
>>> lst2[1] = 42
>>> lst2
[1, 42, 3, 4]
>>> lst1 = lst1 + [5]
>>> lst1 == lst2
False
>>> lst1
[1, 42, 3, 4, 5]
>>> lst2

>>> lst2 is lst1
```


MUTATION

```
>>> lst1 = [1, 2, 3]
>>> lst2 = [1, 2, 3]
#compares each value
>>> lst1 == lst2
True
#compares references
>>> lst1 is lst2
False
>>> lst2 = lst1
>>> lst2 is lst1
True
>>> lst1.append(4)
>>> lst1
[1, 2, 3, 4]
>>> lst2
[1, 2, 3, 4]
```

```
>>> lst2[1] = 42
>>> lst2
[1, 42, 3, 4]
>>> lst1 = lst1 + [5]
>>> lst1 == lst2
False
>>> lst1
[1, 42, 3, 4, 5]
>>> lst2
[1, 42, 3, 4]
>>> lst2 is lst1
```


MUTATION

```
>>> lst1 = [1, 2, 3]
>>> lst2 = [1, 2, 3]
#compares each value
>>> lst1 == lst2
True
#compares references
>>> lst1 is lst2
False
>>> lst2 = lst1
>>> lst2 is lst1
True
>>> lst1.append(4)
>>> lst1
[1, 2, 3, 4]
>>> lst2
[1, 2, 3, 4]
```

```
>>> lst2[1] = 42
>>> lst2
[1, 42, 3, 4]
>>> lst1 = lst1 + [5]
>>> lst1 == lst2
False
>>> lst1
[1, 42, 3, 4, 5]
>>> lst2
[1, 42, 3, 4]
>>> lst2 is lst1
False
```


MUTATION

Write a function that removes all instances of an element from a list.

```
def remove_all(el, lst):  
    """  
    >>> x = [3, 1, 2, 1, 5, 1, 1, 7]  
    >>> remove_all(1, x)  
    >>> x  
    [3, 2, 5, 7]  
    """
```


MUTATION

Write a function that removes all instances of an element from a list.

```
def remove_all(el, lst):  
    """  
    >>> x = [3, 1, 2, 1, 5, 1, 1, 7]  
    >>> remove_all(1, x)  
    >>> x  
    [3, 2, 5, 7]  
    """  
    while el in lst:  
        lst.remove(el)
```


MUTATION

Write a function that takes two values `x1` and `el`, and a list, and adds as many `el`'s to the end of this lists there are `x`'s.

```
def add_this_many(x, el, lst):  
    """ Adds el to the end of lst the number of times x occurs  
    in lst.  
    >>> lst = [1, 2, 4, 2, 1]  
    >>> add_this_many(1, 5, lst)  
    >>> lst  
    [1, 2, 4, 2, 1, 5, 5]  
    >>> add_this_many(2, 2, lst)  
    >>> lst  
    [1, 2, 4, 2, 1, 5, 5, 2, 2]  
    """
```


MUTATION

Write a function that takes two values `x1` and `el`, and a list, and adds as many `el`'s to the end of this lists there are `x`'s.

```
def add_this_many(x, el, lst):
    """ Adds el to the end of lst the number of times x occurs
    in lst.
    >>> lst = [1, 2, 4, 2, 1]
    >>> add_this_many(1, 5, lst)
    >>> lst
    [1, 2, 4, 2, 1, 5, 5]
    >>> add_this_many(2, 2, lst)
    >>> lst
    [1, 2, 4, 2, 1, 5, 5, 2, 2]
    """

    count = 0
    for element in lst:
        if element == x:
            count += 1
    while count > 0:
        lst.append(el)
        count -= 1
```


MUTATION

Write a function that takes two values `x1` and `el`, and a list, and adds as many `el`'s to the end of this lists there are `x`'s.

```
def add_this_many(x, el, lst):
```

```
    """ Adds el to the end of lst the number of times x occurs
    in lst.
```

```
    >>> lst = [1, 2, 4, 2, 1]
```

```
    >>> add_this_many(1, 5, lst)
```

```
    >>> lst
```

```
    [1, 2, 4, 2, 1, 5, 5]
```

```
    >>> add_this_many(2, 2, lst)
```

```
    >>> lst
```

```
    [1, 2, 4, 2, 1, 5, 5, 2, 2]
```

```
    """
```

```
    count = 0
```

```
    for element in lst:
```

```
        if element == x:
```

```
            count += 1
```

```
    while count > 0:
```

```
        lst.append(el)
```

```
        count -= 1
```

```
    for el in lst:
        if el == x:
            lst.append(el)
```

Wrong solutions because the elements are added to the end of the list as you iterate. Thus it could be iterating for ever.

```
add_this_many(2, 2, lst)
```


ORDERS OF GROWTH

- When we have really large inputs, we need to worry about efficiency.
- We measure efficiency by runtime (Time complexity).
- How long does the functions take to run in terms of the size of the input?
- If the size of the input grows, how does the runtime change?

ORDERS OF GROWTH

- We use Big- Θ notation means a tight bound on time complexity.
- $\Theta(n^2)$ means that the function's runtime is no larger and no smaller than quadratic of the input.

ORDERS OF GROWTH

- n : size of problem
- $R(n)$: amount of resource used (time or space)
- $R(n) = \Theta(f(n))$
- $k_1 * f(n) \leq R(n) \leq k_2 * f(n)$
- where k_1 and k_2 are some constants and $k_1 \leq k_2$
- Assume n is larger than some minimum m

ORDERS OF GROWTH

```
def square(n):  
    return n * n
```

1 primitive operation *
For our purposes, * is constant time

input	function call	number of	number of operations
1	square(1)	1*1	1
2	square(2)	2*2	1
...
100	square(100)	100*100	1
...
n	square(n)	n*n	1

ORDERS OF GROWTH

```
def square(n):  
    return n * n
```

1 primitive operation *
For our purposes, * is constant time

$\Theta(1)$

input	function call	number of	number of operations
1	square(1)	1*1	1
2	square(2)	2*2	1
...
100	square(100)	100*100	1
...
n	square(n)	n*n	1

ORDERS OF GROWTH

```
def factorial(n):  
    if n == 0:  
        return 1  
    return n * factorial(n - 1)
```

Each recursive call has
a constant amount operations.
But we have n recursive calls

input	function call	return value	number of operations
1	factorial(1)	1*1	1
2	factorial(2)	2*1*1	2
...
100	factorial(100)	100*99*...*1*1	100
...
n	factorial(n)	n*(n-1)*...*1*1	n

ORDERS OF GROWTH

```
def factorial(n):  
    if n == 0:  
        return 1  
    return n * factorial(n - 1)
```

Each recursive call has
a constant amount operations.
But we have n recursive calls

$\Theta(n)$

input	function call	return value	number of operations
1	factorial(1)	1*1	1
2	factorial(2)	2*1*1	2
...
100	factorial(100)	100*99*...*1*1	100
...
n	factorial(n)	n*(n-1)*...*1*1	n

ORDERS OF GROWTH

- $\Theta(1)$ - constant time; same time regardless of input size.
- $\Theta(\log n)$ - logarithmic time; e.g. usually dividing the problem down by some factor.
- $\Theta(n)$ - linear time; e.g. usually 1 loop
- $\Theta(n^2)$, $\Theta(n^3)$, etc - polynomial time; e.g. nested loops
- $\Theta(c^n)$ - exponential time; where c is some constant; really horrible time complexity; e.g. tree recursion

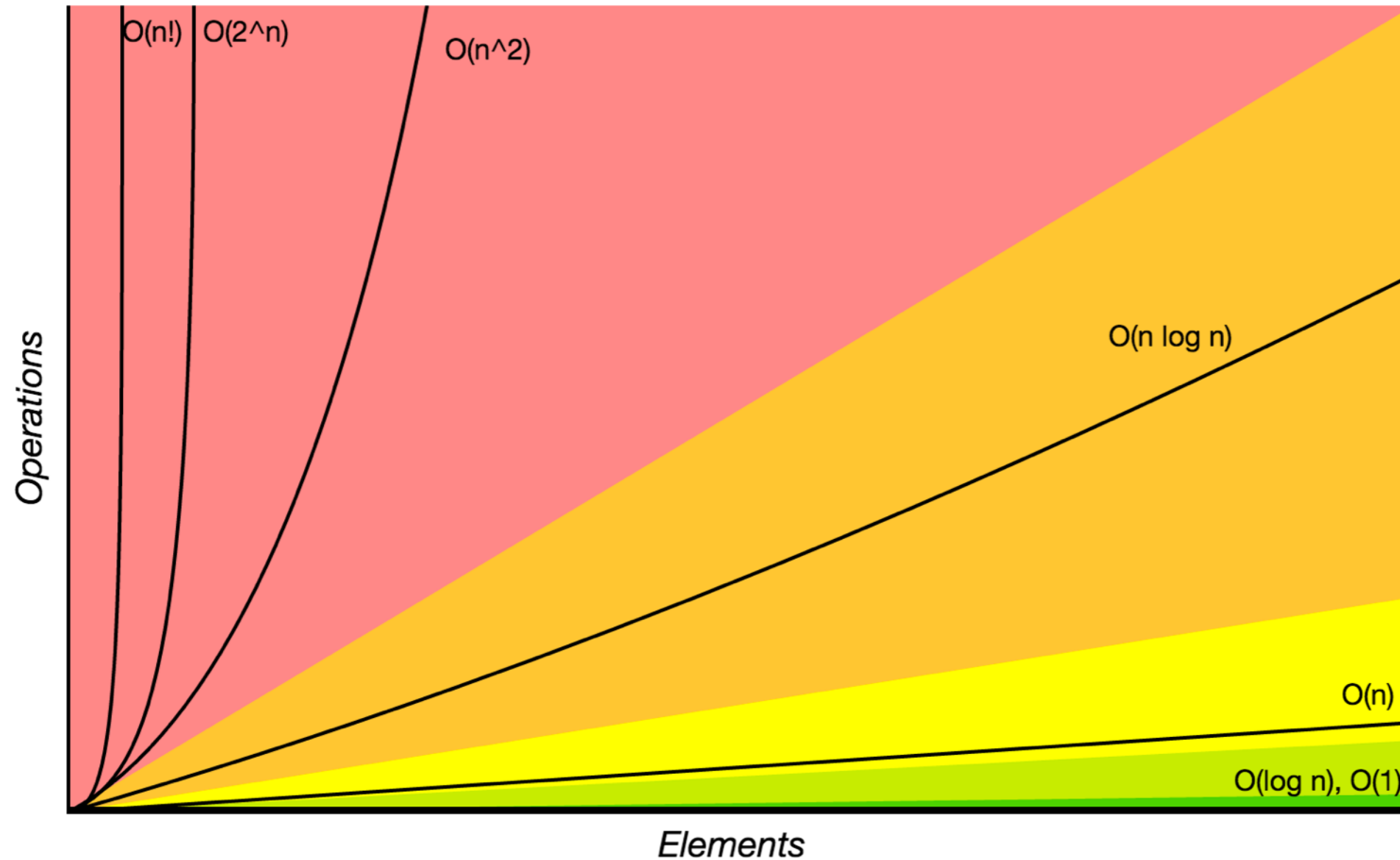
ORDERS OF GROWTH

- Constant time is the best and exponential is the worse.
- Any polynomial is worse than any logarithmic.
- Higher degree polynomial worse than lower degree.

ORDERS OF GROWTH

Big-O Complexity Chart

Horrible Bad Fair Good Excellent



For this class,
assume O is Θ .

See appendix,
for other runtime
notation.

ORDERS OF GROWTH

- Since we care about the runtime when n gets infinitely large, we can drop lower order terms and constants.
 - $\Theta(2n^3 + 6n + \log(n)) = \Theta(n^3)$
- Should always provide the tightest bound.

ORDERS OF GROWTH

- Count the number of iterations and/or recursive calls.
- Find the number of operations per iteration or recursive call.
- Nested loops are usually some polynomial time.
- Exponential time are usually tree recursive.
- Beware of **return** statements because it exits out of a frame before the loops are finished.

NONLOCAL

- We could only access variables in parent frames and not modify them.
- Nonlocal allows us to modify variables in parents frame and outside of the current frame.

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```
def stepper(num):  
    def step():  
        num = num + 1  
        return num  
    return step
```


NONLOCAL

- We could only access variables in parent frames and not modify them.
- Nonlocal allows us to modify variables in parents frame and outside of the current frame.

```
def stepper(num):  
    def step():  
        num = num + 1  
        return num  
    return step
```

Error: We are trying to use num before we assigned it

NONLOCAL

- We could only access variables in parent frames and not modify them.
- Nonlocal allows us to modify variables in parents frame and outside of the current frame.

```
def stepper(num):  
    def step():  
        nonlocal num  
        num = num + 1  
        return num  
    return step
```

In `step`'s frame, does not try to find `num` in local frame.

NONLOCAL

- We could only access variables in parent frames and not modify them.
- Nonlocal allows us to modify variables in parents frame and outside of the current frame.

```
def stepper(num):  
    def step():  
        nonlocal num  
        num = num + 1  
        return num  
    return step
```

For environment diagrams,
num is not a variable in any
of step's frames.

NONLOCAL

- Variables in the global frame cannot be modified using **nonlocal**.
- Variables in the current frame cannot be overridden using **nonlocal**.
- Cannot have a local and nonlocal variable with the same names in a single frame.

```
def stepper(num):  
    def step():  
        nonlocal num  
        num = num + 1  
        return num  
    return step
```


NONLOCAL

- What is wrong with the following code?

```
a = 5
def another_add_one():
    nonlocal a
    a += 1
another_add_one()
```


NONLOCAL

- What is wrong with the following code?

```
a = 5
def another_add_one():
    nonlocal a
    a += 1
another_add_one()
```

a is a variable in the global frame.

Nonlocal cannot be used to modify variables in the global frame.

NONLOCAL

- What is wrong with the following code?

```
def adder(x):  
    def add(y):  
        nonlocal x, y  
        x += y  
        return x  
    return add  
adder(2)(3)
```


NONLOCAL

- What is wrong with the following code?

```
def adder(x):  
    def add(y):  
        nonlocal x, y  
        x += y  
        return x  
    return add  
adder(2)(3)
```

y does not exist in any parent frames.
It is a local variable

NONLOCAL

- What is wrong with the following code?

```
def adder(x):  
    z = 5  
    def add(y):  
        z = 8  
        nonlocal x, z  
        x += z  
        return x  
    return add  
adder(2)(3)
```


RECAP

- Lists and dictionaries are mutable. Tuples and strings are immutable.
- Python list objects are references with pointers. When calling functions that takes a list, we pass in the reference (or pointer) and not create a new list.
- Orders of growth tells us how long the running time of the function approach as n approach infinity.
- Constant is better than logarithmic, which is better than polynomial, which is better than exponential.
- Lower polynomial is better than higher polynomial.
- Try drawing a call stack or tree to count the # of operations.
- Nonlocal allows modifying variables not in local frame.

APPENDIX

- Other Runtime notation
- Dictionaries

ORDERS OF GROWTH

- n : size of problem
- $R(n)$: amount of resource used (time or space)
- $R(n) = \Theta(f(n))$
- $k_1 * f(n) \leq R(n) \leq k_2 * f(n)$
- Assume n is larger than some minimum m

ORDERS OF GROWTH

- n : size of problem
- $R(n)$: amount of resource used (time or space)
- $R(n) = \Omega(f(n))$
- $k_1 * f(n) \leq R(n)$
- Assume n is larger than some minimum m

ORDERS OF GROWTH

- n : size of problem
- $R(n)$: amount of resource used (time or space)
- $R(n) = O(f(n))$
- $R(n) \leq k_2 * f(n)$
- where k_1 and k_2 are some constants and $k_1 \leq k_2$
- Assume n is larger than some minimum m

ORDERS OF GROWTH

- $\Omega(f(n))$ is a lower bound.
- $O(f(n))$ is an upper bound.
- $\Theta(f(n))$ is a tight bound because it is both a lower bound and an upper bound.
- Factorial is $O(n^2)$ and $O(n)$. But the tightest bound is $O(n^2)$.

DICTIONARIES

- Dictionaries map keys to values.
- Python dictionaries are unordered.
- We can obtain a key's mapped value by indexing into the dictionary via the key.
- We can add key-value pairs anytime and can also replace a key's value with something else.

DICTIONARIES

- A dictionary key can be any immutable value.
- If we try to place an entry with a mutable key (i.e. list), we will get an unhashable type error.
- We can check whether a dictionary contains a key with in.
- However, to check if a dictionary contains a value, need to iterate through the dictionary

DICTIONARIES

```
>>> numerals = {"I" : 1, "II" : 2, "III" : 3}
>>> numerals["II"]
2
>>> numerals["IV"] = 4
>>> numerals
{"I" : 1, "II" : 2, "III" : 3, "IV" : 4}
>>> numerals["I"] = 100
>>> numerals
{"I" : 100, "II" : 2, "III" : 3, "IV" : 4}
>>> "I" in numerals
True
>>> 100 in numerals
False
```


DICTIONARIES

```
a = {"a":1, "b":2, "c":3, "d":4}
del a["a"]
{"b":2, "c":3, "d":4}
a.pop("d")
4
{"b":2, "c":3}
```

```
for key in dictionary
```

```
for key in dictionary.keys()
```

```
for value in dictionary.values()
```

```
for key, value in dictionary.items()
```


DICTIONARIES

- We can iterate over a dictionary's keys.

```
for key in dictionary
```

```
for key in dictionary.keys()
```

- We can iterate over a dictionary's values.

```
for value in dictionary.values()
```

- We can iterate over a dictionary's keys and values at the same time.

```
for key, value in dictionary.items()
```


DICTIONARIES

- We can delete a dictionary's key-value pair with `del`.

```
a = {"a":1, "b":2, "c":3, "d":4}
del a["a"]
{"b":2, "c":3, "d":4}
```

- We can delete a key and return its value with `pop`.

```
a.pop("d")
4
{"b":2, "c":3}
```